FM 262 W 1-2:50 in GIRV 2110
PART 2 Introduction to superprocesses
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In this second part we cover the introductory theory of superprocesses that are measure-valued stochastic processes. First we study branching Brownian motions and supper Brownian motions via martingale problems. Then we study more general classes of superprocesses. The study of such processes is mostly applied to Mathematical Population Genetics but we explore some possible applications to Mathematical Finance.

1. Martingale problems. (Lecture 5)

   A probability distribution of a random variable is determined by the corresponding probability distribution function. We know well about famous distributions such as Gaussian and Poisson distributions, however, the observed distributions in the financial market or in other fields do not necessarily belong to those well-known distributions. The martingale formulation is quite useful in classifying classes of distributions of stochastic processes. We discuss the basic techniques.

2. Branching diffusions and exit measure. (Lecture 6)

   Consider a system of particles that diffuse on some domain and reproduce new particles according to a branching mechanism. Such system can be seen as a measure-valued process. The branching mechanism creates some discontinuity and additional dynamics in the system. This is a departure from the diffusion theory. Starting with branching Brownian motion as an example, we characterize the system of such processes in terms of martingale problems, branching exit measures and partial differential equations. This theory has been studied extensively, since Kolmogorov suggested a model of branching with a continuous time parameter around 1947. There will be possible applications to the analysis of Monte Calro simulations and the Mathematical Finance.

3. Superprocesses. (Lecture 7-8)

   Superprocesses are generalizations and continuous counterparts of the above particles systems. Two important examples, Dowson-Watanabe superprocess and Fleming-Viot superprocess are considered. These processes arise to model dynamics of large population (its total population and spatial location of population) as scaling limits of the measurer-valued branching processes. Most applications have been done in Mathematical Population Genetics. However, there are potentially many applications to Spatial Studies, because the location parameter is not restricted to the Euclidian space. Now there are several ways of understanding superprocesses. We study qualitative behavior and some representations of them, such as the Le Gall's construction through Brownian snakes.

References:

1. "An Introduction to Superprocesses" by Alison M. Etheridge (2000).

2. "Diffusions, Superprocesses and Partial Differential Equations" by E.B. Dynkin (2002).