Derivatives in Financial Markets with Stochastic Volatility

ERRATUM: CHAPTER 8

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The calculation on page 128 is not correct and should be replaced by the following. We thank Matthew Yandle for pointing this out to us.

The solution to (8.5) can be reduced to a one-dimensional integral that can be speedily computed numerically as follows.

The correction to the barrier price $\widetilde{P_1}(t,x)$ satisfies

$$\begin{aligned}\mathcal{L}_{BS}(\bar{\sigma})\widetilde{P_1} &= \mathcal{A}P_0 \qquad x > B,\, t < T\\ \widetilde{P_1}(T,x) &= 0\\ \widetilde{P_1}(t,B) &= 0.\end{aligned}$$

Let

$$u(t,x) = \widetilde{P_1} + (T-t)\mathcal{A}P_0,$$

for $x \geq B$. Then $u(t,x)$ solves the simpler problem

$$\begin{aligned}\mathcal{L}_{BS}(\bar{\sigma})u &= 0 \qquad x > B,\, t < T\\ u(T,x) &= 0\\ u(t,B) &= g(t),\end{aligned}$$

where we define

$$g(t) = (T-t)\mathcal{A}P_0\,|_{x=B^+} = (T-t)\lim_{x\to B^+}\left(V_2x^2\frac{\partial P_0}{\partial x^2} + V_3x^3\frac{\partial P_0}{\partial x^3}\right)(t,x),$$

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meaning the partial derivatives taken into the "in" region to the right of the barrier. In fact $g(t) = (T-t)F(t,B)$ where $F(t,x)$ is given by equation (8.6) on page 127.

Then the problem can be transformed to the constant coefficient backward heat equation by the simple transformations

$$\begin{aligned} z &= \log x \\ u(t,x) &= \exp\left(\left(-\frac{1}{2\bar{\sigma}^2}(r-\frac{1}{2}\bar{\sigma}^2)^2 - r\right)(T-t) - \frac{1}{\bar{\sigma}^2}(r-\frac{1}{2}\bar{\sigma}^2)z\right) v(t,z). \end{aligned}$$

Defining $L = \log B$, $v(t,z)$ then solves

$$\begin{aligned} \frac{\partial v}{\partial t} + \frac{1}{2}\bar{\sigma}^2\frac{\partial^2 v}{\partial z^2} &= 0, \qquad z > L,\ t < T \\ v(T,z) &= 0 \\ v(t,L) &= \tilde{g}(t), \end{aligned}$$

where

$$\tilde{g}(t) = \exp\left(\left(\frac{1}{2\bar{\sigma}^2}(r-\frac{1}{2}\bar{\sigma}^2)^2 + r\right)(T-t)\right) B^{r/\bar{\sigma}^2-\frac{1}{2}} g(t).$$

The probabilistic representation of v is simply

$$v(t,z) = \mathbb{E}\{\tilde{g}(\tau)\mathbf{1}_{\{\tau\le T\}} \mid B_t = z > L\},$$

where (B_t) is a Brownian motion with $< B >_t = \bar{\sigma}^2 t$, and τ is the first time after t that it hits L. Using the distribution of the hitting time τ (see, for example, Karatzas and Shreve "Brownian Motion and Stochastic Calculus" proposition 8.5, Chapter 2), the solution to this is given by the integral

$$v(t,z) = \frac{1}{\bar{\sigma}\sqrt{2\pi}}\int_t^T \frac{(z-L)}{(s-t)^{3/2}} e^{-(z-L)^2/2\bar{\sigma}^2(s-t)}\tilde{g}(s)\,ds.$$

From this, we obtain the correction to the barrier price

$$\widetilde{P_1}(t,x) = -(T-t)\mathcal{A}P_0 + x^{-\frac{1}{\bar{\sigma}^2}(r-\frac{1}{2}\bar{\sigma}^2)} e^{(-\frac{1}{2\bar{\sigma}^2}(r-\frac{1}{2}\bar{\sigma}^2)^2-r)(T-t)} v(t,\log x).$$